

# Geometric Morphometric Analysis of Projectile Points from the Southwest United States

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## 1 **Abstract**

2 Traditional analyses of projectile points often use visual identification, the presence or absence of discrete  
3 characteristics, or linear measurements and angles to classify points into distinct types. Geometric mor-  
4 phometrics provides additional tools for analyzing, visualizing, and comparing projectile point morphology  
5 utilizing the whole or parts of the form in either two or three dimensions. This study is an analysis of the  
6 effectiveness of geometric morphometric methods for identifying technological similarity in 2D projectile  
7 point outlines for previously classified late prehistoric projectile points found in the U.S. Southwest and  
8 unclassified projectile points from Tonto Basin, Arizona. Various methods from geometric morphometrics  
9 were compared to determine which method best reproduced the original classification scheme. Elliptical  
10 Fourier analysis was compared with various configurations of semilandmark and landmark analyses using  
11 generalized Procrustes analysis. These methods were applied to the complete projectile point form, and  
12 the landmark analysis was also applied to half of the lower quadrant of the projectile point—essentially one  
13 corner of the projectile point. The landmark analysis applied to the corner of the projectile point provided  
14 the best results. This method was then applied to the Tonto Basin points. Hierarchical clustering was used  
15 on the Tonto Basin projectile point morphometric data to explore the variation in shapes between sites. To  
16 demonstrate that geometric morphometric methods can be used without relying on typologies, a network  
17 analysis of the morphometric distances was also conducted. This network graph produced distinct clusters  
18 of technological similarity in projectile point outlines, while also showing the continuous variation between  
19 points. These results demonstrate the effectiveness of geometric morphometrics for the 2D analysis of late  
20 prehistoric arrow points in the U.S. Southwest.

21  
22 **Keywords:** American Southwest, Hohokam, Arizona, projectile points, lithics, computational archaeology, geometric morphometrics

## 23 Introduction

24 Geometric morphometrics (GM) is a quantitative approach to studying shape in two or three dimensions that  
25 has recently been adopted in archaeology (**MacLeod2017-yl**; **Okumura2019-urShott2010-fn**). It has numer-  
26 ous advantages over traditional lithic analyses, particularly because it can overcome the reliance on linear  
27 dimensions (**Shott2010-fn**). Lithic artifacts can be assigned to typologies or directly compared without the  
28 use of a typology, as will be demonstrated in this [chapter paper](#). There are several approaches within GM  
29 that provide similar results through different methods. One of the more traditional approaches is to place  
30 landmarks at homologous locations around the object. Landmarks can be augmented with semilandmarks,  
31 which are points placed relative to another using a consistent rule—usually equidistant spacing between two  
32 points (**Okumura2019-ur**). Another common approach is to use elliptical Fourier analysis to compare the  
33 outlines of objects. Each method has strengths and weaknesses. A major purpose of this study is to evaluate  
34 the effectiveness of these methods for analyzing projectile points in the U.S. Southwest during the late prehis-  
35 toric period ([after the introduction of the bow and arrows specifically during the Hohokam Classic Period—AD](#)  
36 [1100-1500](#)).

37 Once the method of analyzing the [projectile](#) points has been determined, the next step is to determine how  
38 to compare [projectile](#) points using the results of the analysis. One approach would be to use an existing re-  
39 gional typology and to assign projectile points to the closest match (**Kocer2017-au**). Another approach, would  
40 be to use cluster analysis to assign [projectile](#) points to newly created types [e.g., [Petrik2018-pd](#); [Matzig2021-id](#)].  
41 The final approach would be to ignore typologies and compare the morphometric distance for each projec-  
42 tile point directly. [This is the second primary purpose of this study—to evaluate the effectiveness of these](#)  
43 [approaches for use in analyzing projectile points from the Southwest United States.](#)

44 Regional analyses are fundamental parts of archaeology, but there are many challenges to overcome. One  
45 of these challenges is harmonizing the different categorization schemes (i.e., ontologies) used throughout the  
46 region. Another of these challenges, is determining whether the current categories are useful. The U.S. South-  
47 west has a long history of regional ceramic typologies (**Colton1956-zy**; **gladwin1930a**; **Hargrave1932-ng**;  
48 **Kidder1915-ae**; **Martin1940-jg**), but there are still disagreements, challenges, and competing definitions  
49 (**Duff1996-au**). Regional analyses in the Southwest, based in large part on pottery, have produced many  
50 useful insights (**Bernardini2005-ue**; **Clark2019-bz**); **Hegmon2016-xw**; **Mills2013-wq**; **Peoples2018-ib**). How-  
51 ever, one type of material culture that has received little attention—in the Southwest at least—is lithics (i.e.,  
52 chipped stone). Projectile points are commonly discussed during the archaic period of the Southwest, and  
53 they are common topics in many other areas of the North American continent and world where they are  
54 found, but they are rarely discussed after the appearance of pottery.

55 Despite the over-emphasis on pottery in the Southwest, there are some excellent resources on projectile  
56 point typologies (**Hoffman1997-hb**; **Justice2002-cf**; **Loendorf2004-tp**); **Sliva2006-nq**]. However, ad hoc ap-  
57 proaches are common, and these cannot [easily](#) be extrapolated beyond [a-specific-project-specific projects](#).  
58 Even using existing resources can make comparisons difficult. How does Tagg's (**Tagg1994-wi**) Type 23 com-  
59 pare to Sliva's (**Sliva2006-nq**) Cohonina Side-notched? There is an answer, but often it is easier to come up  
60 with a new typology schema than to try to harmonize existing work.

61 Another challenge that is not unique to projectile points is that interpretations may differ between analysts.  
62 Exactly when does a base begin curving enough to be called basal notched? Even the difference between a  
63 side-notched and a corner-notched point can, at times, be ambiguous. Not to mention the frustrating situation  
64 where a point appears to have one corner-notch and one side-notch. How should one place this point into  
65 an existing typology? These are questions that can be handled in different ways that differ from analyst to  
66 analyst. Idiosyncrasies and biases are impossible to be rid of entirely, but using approaches such as [the those](#)  
67 [described here those described in this paper](#) can reduce them and increase the reproducibility of the process.

68 By necessity, this [chapter paper](#) covers a number of topics. The geographic area is the U.S. Southwest,  
69 but the methods and analysis are applicable to any area. The [code and data used are available for reuse](#)  
70 [and modification.](#) The primary purpose was to explore geometric morphometric methods using previously

71 typed specimens from the Southwest and untyped specimens from the Tonto Basin. ~~As mentioned, the~~  
72 ~~corner-based landmark analysis proved most successful.~~ Another purpose was to analyze the results with  
73 and without using typologies. The results demonstrate that ~~both approaches are useful,~~ in this particular  
74 case, a combined landmark and semilandmark approach is most effective and that useful analyses can be  
75 conducted with and without the use of typologies.

## 76 Background

77 In order to test the effectiveness of geometric morphometric methods, I needed a dataset of well-typed  
78 projectile points that could be used as a validation set. I chose to use the typology published by Noel Justice  
79 (**Justice2002-cf**) for the simple reason that it is easily accessible and contains numerous illustrations. These  
80 illustrations were used as type specimens to compare projectile points from Tonto Basin in central Arizona  
81 (Figure 1). These points were excavated in a series of large cultural resource management projects necessi-  
82 tated by work on the Roosevelt Dam. The largest project—the Roosevelt Platform Mound Study—included 129  
83 sites. Most of the sites date between AD 1275 and 1325 with occupation continuing until around AD 1450  
84 (**Rice1998-ku**). In the original analysis, ~~Projectile~~ projectile points were classified according to small and large  
85 points and then subdivided based on morphological characteristics (**Rice1994-rk**). The typology used is an  
86 excellent demonstration of the difficulty in conducting projectile point studies in this area, as the typology is  
87 idiosyncratic to this specific project, and cannot be easily compared with other datasets. This is not a criticism  
88 of the analyst's choice to create a new typology, as no existing typology met the needs of the researchers.

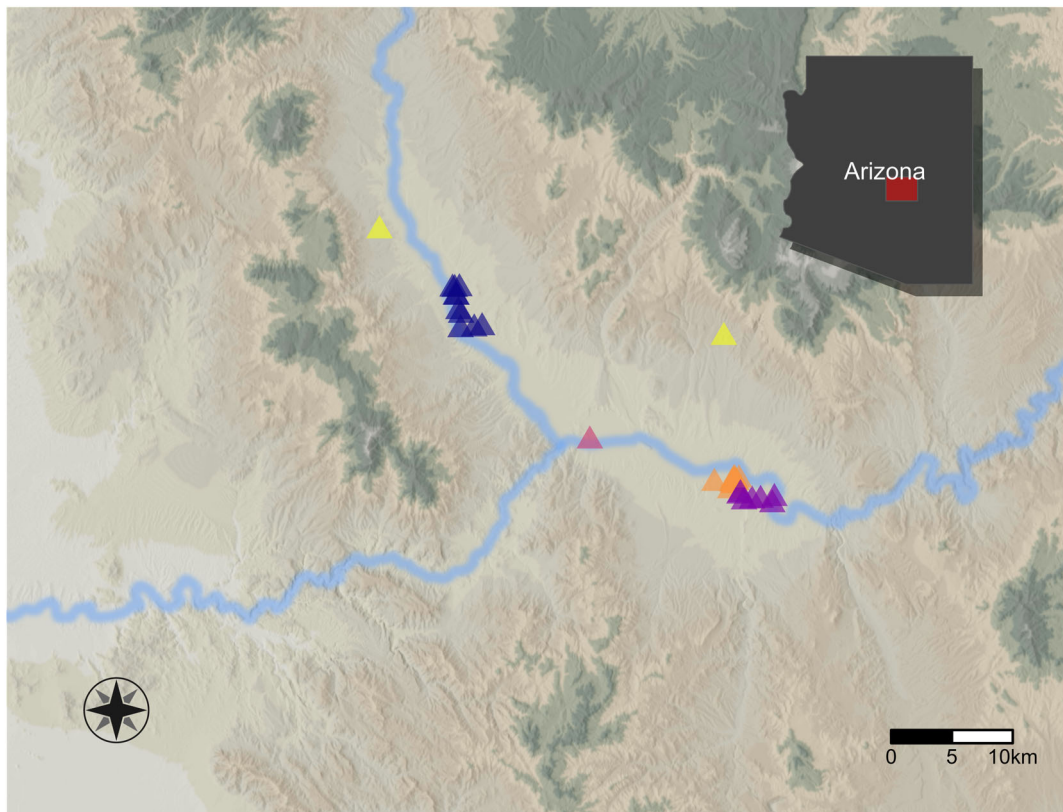
89 This is an exploratory analysis designed to minimize the amount of time spent collecting data and to be as  
90 reproducible as possible. There are a number of research steps that are often not addressed in publications.  
91 This missing documentation can make reproducing results challenging. I will describe the rationale for relevant  
92 decisions, but the script used to generate the analysis will be included in an RMarkdown document in the  
93 supplemental material (see statement at end of manuscript).

94 One of the key elements of this study is reproducibility, which necessitates automation. Projectile point  
95 analysis often includes assigning a point to a type based on linear metrics—sometimes angular measurements  
96 as well, and the presence or absence of various features (e.g., concave base, serrated blades, corner-notches).  
97 But often, the analyst is left to visually compare the point to various type specimens to identify the closest  
98 match. ~~In my experience, this can be a frustrating way to spend your time.~~ This method is harder to reproduce  
99 and subject to greater human error. Yet, algorithms are only part of the answer, and human judgment and  
100 context are still critical to any analysis. The key is to minimize the possibilities for error and maximize the  
101 opportunities for reproducibility, which I have tried to do here. Thus, one of the key questions of this research  
102 is to determine what input should be left to the analyst and what can be left to automated or standardized  
103 procedures.

## 104 Data Collection

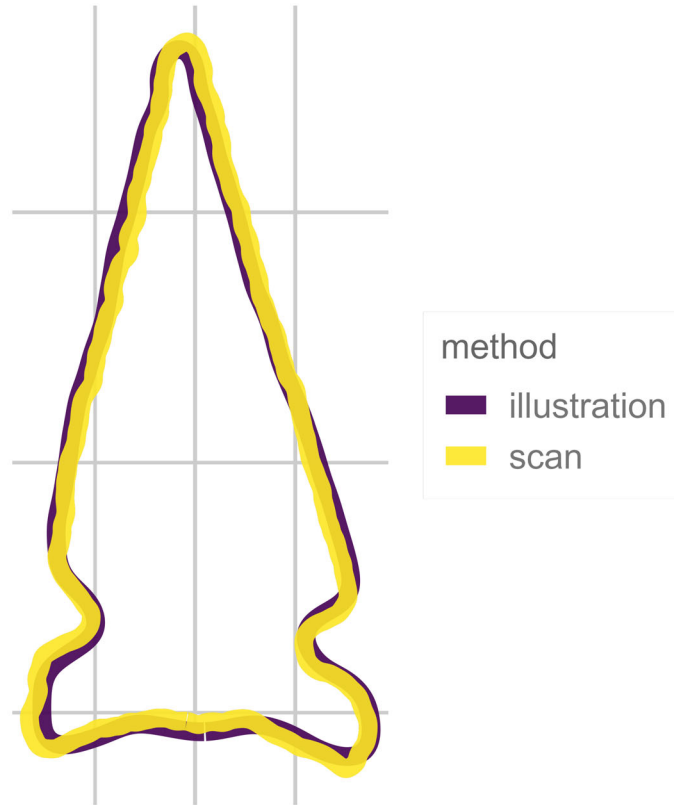
105 This study has two sources of data: illustrations of projectile points published by Justice (**Justice2002-cf**) and  
106 images of projectile points from collection-collections held at Arizona State University. The datasets include  
107 74 illustrations from Justice's publication and 90 projectile points from Tonto Basin. ~~Many~~ The 74 illustrations  
108 do not include all of Justice's ~~types~~ illustrations or types, as many could not be included because there were  
109 so few complete, illustrated examples.

110 It is worthwhile to question how an illustration compares to an image of a physical projectile point obtained  
111 from a flatbed scanner. Fortunately, illustrations have been published for some of the projectile points in this  
112 study. Figure 2 is a comparison of outlines created from an illustration and scan of the same projectile point.  
113 There are subtle differences between the two mediums—the base is slightly more rounded in places than the  
114 scan. These differences are detectable in a morphometrics analysis, however, the differences are minor as



Area ▲ Cline Mesa ▲ Livingston ▲ Rock Island ▲ Schoolhouse Mesa ▲ Uplands

**Figure 1.** Location of Tonto Basin in the state of Arizona, United States, [along with archaeological sites discussed in this paper grouped by site cluster.](#)



**Figure 2.** Comparison of projectile point outlines for an illustration and a scan of the same projectile point (Oliver and Simon 1997: Figure 9.3; Specimen 33598)

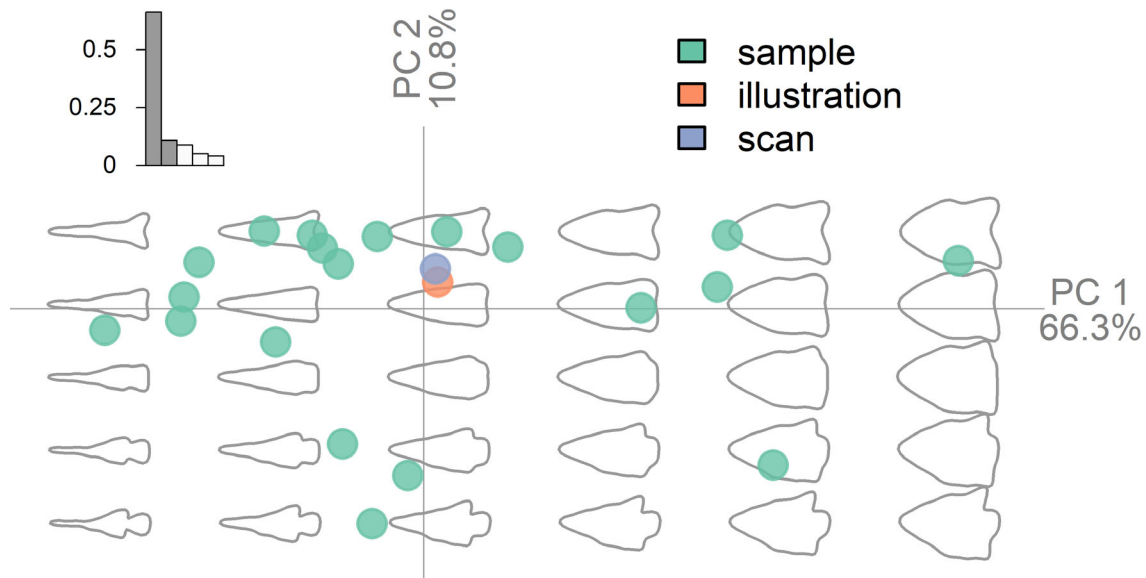
115 seen in figure 3. [These minor differences would not affect the results of the analyses presented here or most](#)  
 116 [GM analyses.](#) The quality of projectile point illustration can vary, but from this brief comparison there should  
 117 be no hesitation using illustrations for 2D morphometric analysis.

118 Justice's projectile point illustrations were scanned, and the illustrations were converted into individual,  
 119 solid black outlines and saved as jpeg files using common image-editing software. The open source statistical  
 120 software R was used for all analyses ([R\\_Core\\_Team2022-wb](#)). The Momocs package ([Bonhomme2014-gt](#))  
 121 has an import function to convert jpeg files into outlines. This is a major advantage over manual outlining  
 122 processes used in popular GM software, such as tpsDig ([James\\_Rohlf2015-ui](#)). These outlines form the the  
 123 basis of the geometric morphometric analyses conducted here with the exception of the landmark analyses.  
 124 Landmarking was performed using the tpsDig software. The [Momocs package has that capability, but I have](#)  
 125 [found tpsDig's utility for landmarking to be superior.](#) The Tonto Basin projectile point images were created  
 126 using a flatbed scanner at 1200 DPI <sup>1</sup>. The images were converted to outlines using the same process as the  
 127 projectile point illustrations.

## 128 **Projectile Points of the Southwest**

129 There are a few regional typologies for projectile points in the Southwest: [common examples were authored](#)  
 130 [by primary examples are](#) Hoffman ([Hoffman1997-hb](#)), Justice ([Justice2002-cf](#)), Loendorf and Rice ([Loendorf2004-tp](#)),  
 131 and Sliva ([Sliva2006-nq](#)). Altogether, these four typologies include 129 projectile point types, although many

<sup>1</sup> Many of the images were obtained by the author but some were generously contributed by [Josh Joshua](#) Watts—see the results of his study here: ([Watts2013-ub](#)).



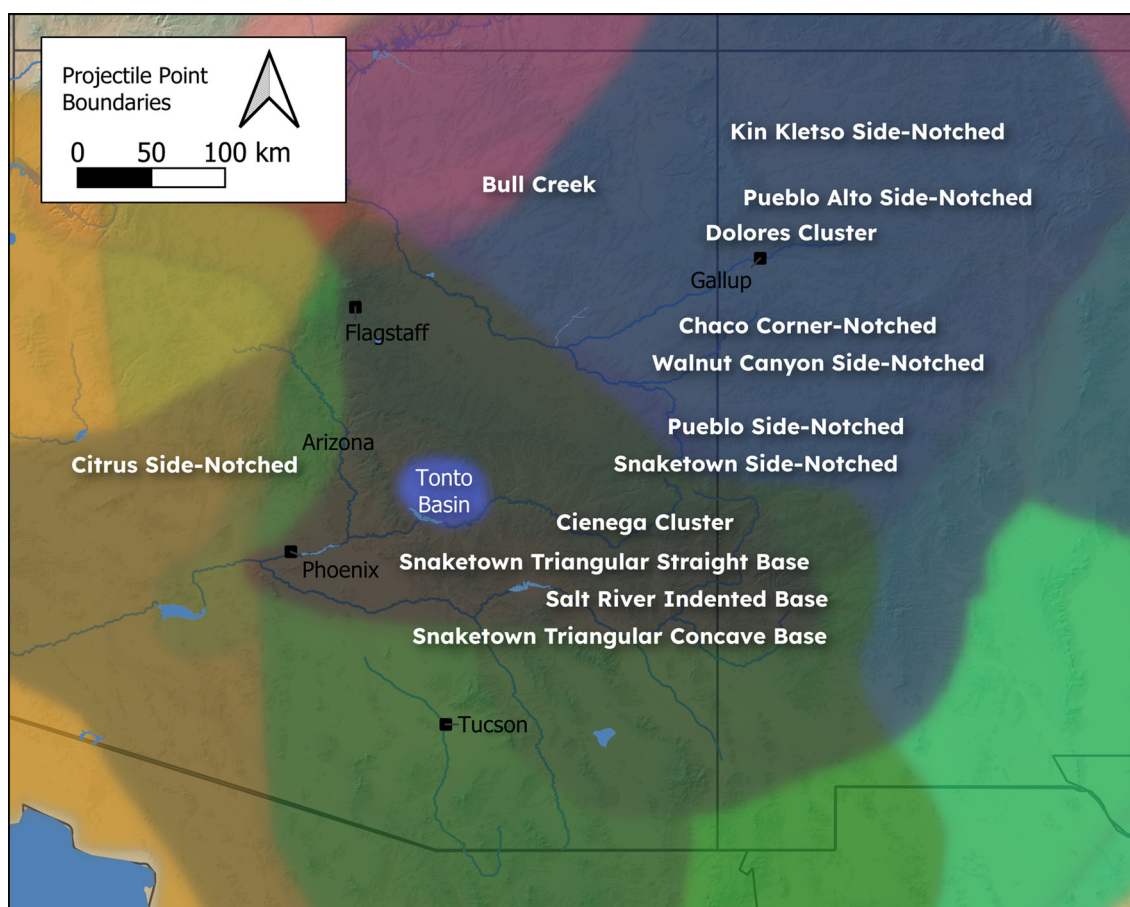
**Figure 3.** Principal component plot comparing the morphometric differences between a sample of 20 random projectile points and the illustration and scan of the same projectile point. [The morphospace is also projected. The morphospace computes the projectile point outlines shown in the figure, which represents how shapes vary along each axis.](#)

132 overlap. In some cases, the authors identify correlates of the types [from other typologies. This allows for](#)  
 133 [some harmonization of the different typologies.](#) Many types date to the Archaic period, and thus predate  
 134 the primary period [we are interested in-I am interested in \(AD 1100-1500\).](#) Not all of the [projectile](#)  
 135 [points](#) were ascribed dates by the authors. Justice lists 23 [projectile](#) points that overlap with the AD 1100-1500  
 136 period (the maximal dates for the Hohokam Classic period). Projectile points have restricted geographical  
 137 boundaries, although these boundaries correspond to much greater areas than ceramic types typically do  
 138 (**Buchanan2019-vn**). Figure 4 shows that several projectile point boundaries defined by Justice overlap with  
 139 the Tonto Basin.

140 For this study, I digitized 74 projectile point images from Justice's publication representing 8 projectile point  
 141 types (table 1). Justice placed each projectile point type into a cluster of related points. Figure 5 shows the  
 142 projectile point outlines by type. The included projectile point types include some Archaic points and types  
 143 not expected to overlap with the Tonto Basin projectile points. [Archaic-Small numbers of archaic](#) points are  
 144 often found at later sites, [and these.](#) [These are likely the result of collecting activities and not indicative of](#)  
 145 [the continued use of these points \(see Justice2002-cf for numerous examples\).](#) [These curated archaic](#) types  
 146 form useful comparisons, as they should not match [to non-archaic](#) projectile points. One limitation of this  
 147 study is that projectile points must be complete, or nearly complete (minor damage to the tip or another part  
 148 of the point that was judged to not significantly impact the shape of the point was ignored). Thus, not all of  
 149 the illustrations included in Justice's book could be included in the outline analyses.

### 150 Tonto Basin Projectile Points

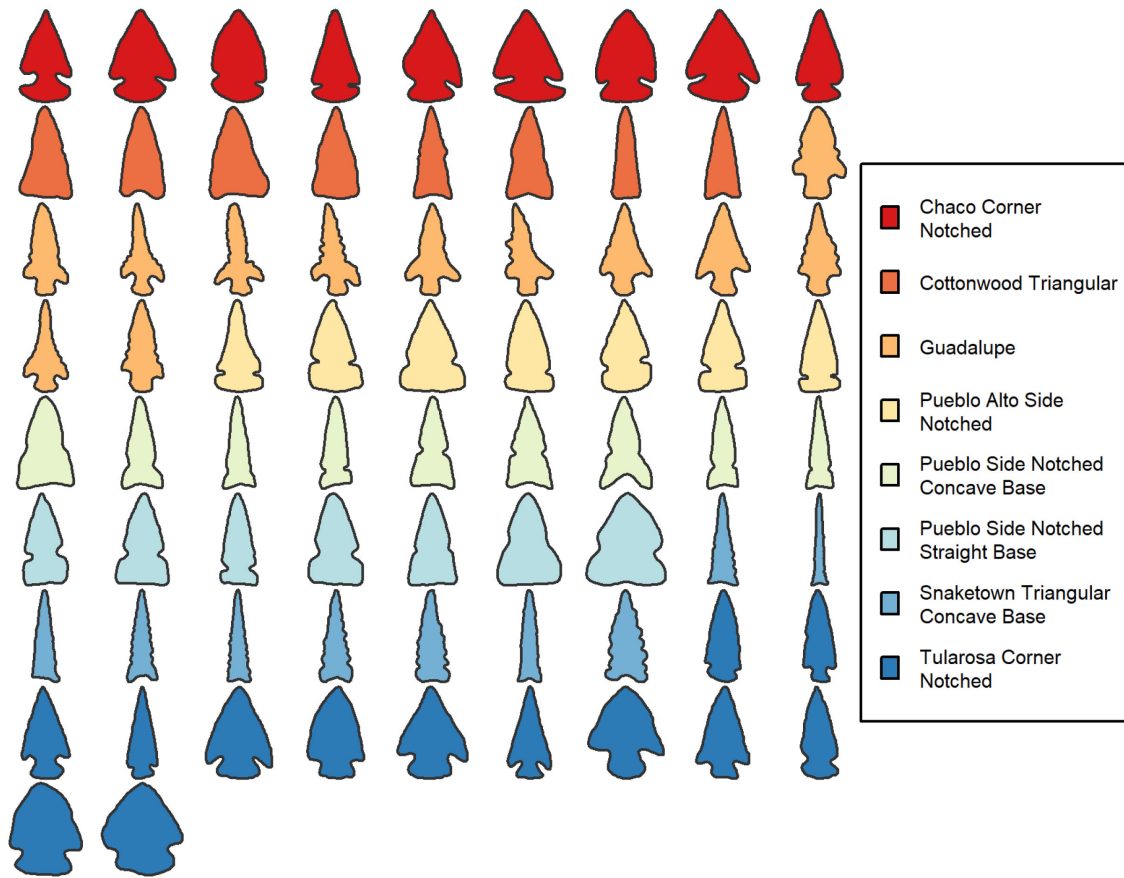
151 The sample of Tonto Basin points used in this study come from the Roosevelt Platform Mound Study. They  
 152 come from 18 different sites that were grouped into five clusters in the original reports (**Rice1998-ku**). Figure  
 153 [6 shows the projectile points-1 shows the sites](#) used in this study grouped by site cluster. The majority of  
 154 these sites were occupied during the Roosevelt phase (AD 1275-1325) and early portion of the Gila phase (AD  
 155 1325-1450). The sites consist primarily of compounds, room blocks, and platform mounds.



**Figure 4.** Location of selected projectile point boundaries defined by Justice (2002) as digitized by Buchanan and colleagues (2019). Darker colors represent greater overlap in the number of projectile point boundaries.

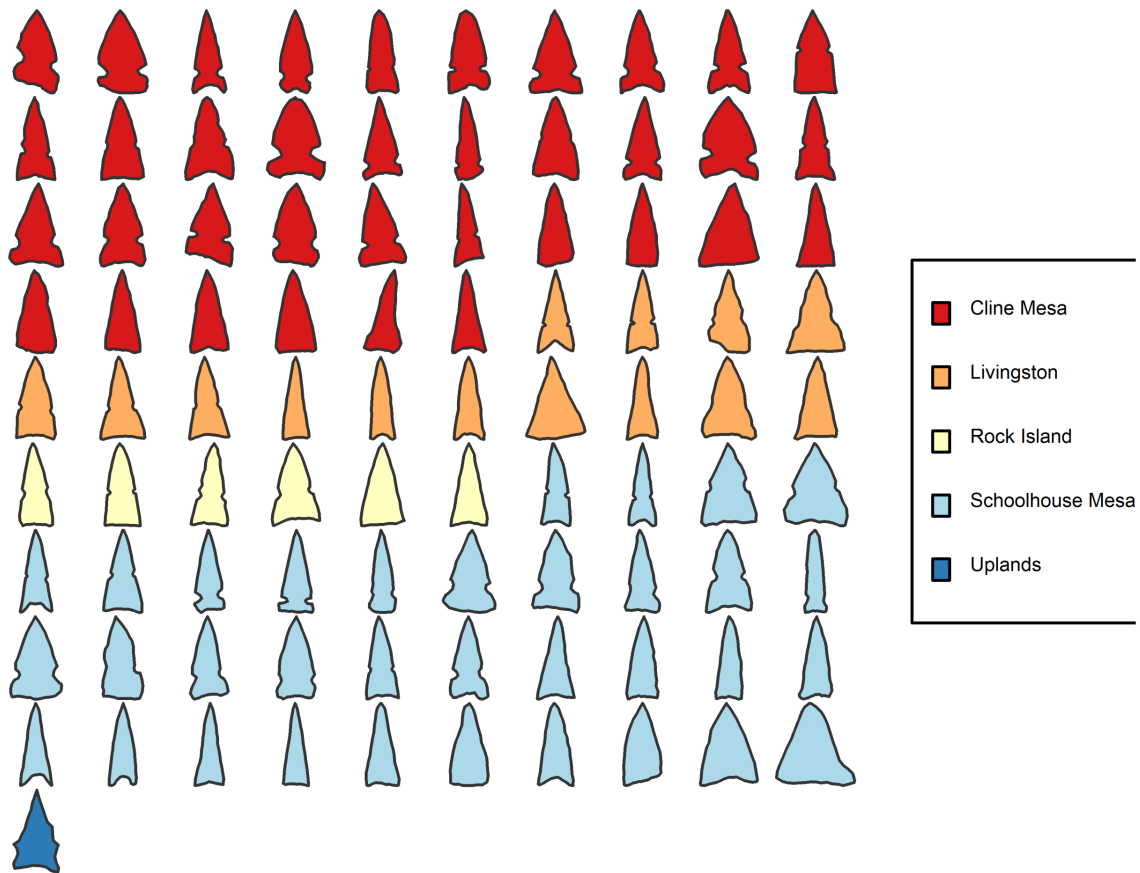
**Table 1.** Cluster Names, Types, and Number of Samples

Cluster	Type	total
Chaco	Chaco Corner Notched	9
Chaco	Pueblo Alto Side Notched	7
Cienega	Tularosa Corner Notched	13
Livermore	Guadalupe	12
Pueblo Side Notched	Pueblo Side Notched Concave Base	9
Pueblo Side Notched	Pueblo Side Notched Straight Base	7
Snaketown	Snaketown Triangular Concave Base	9
Western Triangular	Cottonwood Triangular	8



**Figure 5.** Outlines of projectile point illustrations taken from Justice (2002). Note that the projectile points are not scaled.





**Figure 6.** Outlines of projectile points from sites in the Tonto Basin. Note that the projectile points are not scaled.

156 ~~Archaeology sites from Tonto Basin included in discussion. The original reports grouped each site into~~  
 157 ~~different clusters. Note that two of the platform mound sites are labeled separately.~~

158 The projectile points exhibit a variety of forms (see figure 76). Rice (Rice1994-rk) classified Tonto Basin  
 159 points into small and large complexes (likely equivalent to dart and arrow points), and further classified small  
 160 points into the longer Salado series and the shorter Tonto series. These series were further subdivided using  
 161 a custom classification scheme based on blade, tang, and base shape, as well as notch style. This is a logical  
 162 way to classify the points, but it does not easily lend itself to regional comparisons, as other points were  
 163 not classified in the same way. Nearly all of the points in the original sample consisted of side-notched or  
 164 triangular points. Because the results of the analysis discussed below indicated that analyzing the [projectile](#)  
 165 points by shape was the most logical choice, only the triangular and side-notched points were used in this  
 166 study.

## 167 **Methods**

### 168 **Geometric Morphometrics**

169 I used two GM methods in this analysis: elliptical Fourier analysis (EFA) and full generalized Procrustes align-  
 170 ment (GPA). Something to keep in mind is that GM methods analyze the form of the object separated from size,  
 171 position, and orientation. Real-world measurements such as length and width are not explicitly included in  
 172 these methods, although relative dimensions, such as length to width ratio are captured in the overall form of

173 the object. Measurements such as length and weight can be included in various analyses but are not included  
174 here. The purpose of this study is to determine whether GM methods alone are sufficient to discriminate  
175 between types of projectile points and how they can best be used in the context of the U.S. Southwest.

176 EFA was developed by Kuhl and Giardina (**Kuhl1982-kd**) as a quantitative means for describing a closed  
177 outline. There are a handful of papers that use EFA for lithic studies in archaeology [e.g., [Cardillo2010-ys](#);  
178 [Fox2015-ox](#); [Gingerich2014-cb](#); [Hoggard2019-yw](#); [Iovita2011-nz](#); [Iovita2011-zp](#); [Matzig2021-id](#)]. The math-  
179 ematics behind the method are complex to describe, which is one reason the method has not been adopted  
180 as quickly as it should be (**Caple2017-mk**). Caple and colleagues (**Caple2017-mk**) provide an excellent de-  
181 scription of EFA for non-mathematicians, and the reader is referred to their treatise for more details. For  
182 my purposes, it is enough to know that EFA analysis requires a closed outline and a number of harmonics.  
183 The harmonics can be thought of as ellipses in a time series used to describe the shape of the object. [EFA](#)  
184 [creates a series of coefficients-four for each harmonic \(A,B,C,D\)-which can be used in multivariate statistics.](#)  
185 [Most commonly, principal components analysis \(PCA\) is used to transform the EFA values. The PCA results](#)  
186 [can then be used in distance-based methods such as clustering or even network analysis.](#) Three harmonics  
187 can be used to create an oval shape, and 12 harmonics is sufficient for a complex projectile point outline.  
188 The number of harmonics necessary to capture the outline ~~can be computed. For example, if you wish to~~  
189 ~~reconstruct an outline with 99.9% accuracy, then the exact number of harmonics necessary can be calculated~~  
190 ~~to a certain accuracy can be computed by first calculating the harmonic power~~ using the formula:

$$\text{HarmonicPower}_n = \frac{A_n^2 + B_n^2 + C_n^2 + D_n^2}{2}$$

191 where n is the [harmonic power number of harmonics](#) and A, B, C, and D are the coefficients generated from the  
192 EFA. ~~EFA creates a series of coefficients-four for each harmonic (A, B,C,D)-which can be used in multivariate~~  
193 ~~statistics. Most commonly, principal components analysis (PCA) is used to transform the EFA values. The PCA~~  
194 ~~results can then be used in distance-based methods such as clustering or even network analysis~~ [The harmonic](#)  
195 [power is first calculated for a maximum number of harmonics and then the desired proportion \(e.g., 99%\) of](#)  
196 [the harmonic power can be used as a baseline to determine which number of harmonics has at least that](#)  
197 [much harmonic power.](#)

198 Generalized Procrustes Analysis, or GPA, is primarily a way to align, scale, and rotate ~~points-landmarks~~  
199 (**Gower1975-uv**). Instead of outlines like EFA, GPA requires landmarks located on homologous locations  
200 for each object (**Rohlf1990-mp**). As an alternative to landmarks, semilandmarks can be placed at equidis-  
201 tant locations around the object. [Landmarks and semilandmark approaches can be combined.](#) There is  
202 substantial discussion on the validity of certain types of landmarks and the use of semilandmarks as land-  
203 marks (**De\_Groote2011-mh**; **MacLeod2017-yl**; **Okumura2019-ur**; **Shott2010-fn**). ~~Although there are some~~  
204 ~~methods that can introduce error, there is no reason to exclude semi-landmarks outright.~~ One disadvantage  
205 of traditional landmark analysis, compared to EFA, is that the analyst must be more involved in the selection of  
206 the number and placement of landmarks. Once the landmarks, or semilandmarks are placed on the objects,  
207 they are iteratively modified to achieve the best possible alignment between shapes without changing the rel-  
208 ative positions between landmarks. This modification is done using the GPA procedure. As with EFA, the next  
209 step is usually to perform a PCA analysis. Landmark analysis using GPA is more common, so far, in archaeo-  
210 logical analysis of stone tools than EFA (**Archer2018-zi**; **Bischoff2020-zn**; **Buchanan2015-dx**; **Charlin2018-yg**;  
211 **Fisher2018-jq**; **Gingerich2014-cb**; **Herzlinger2017-ce**; **Lycett2010-od**; **Riede2019-gb**; **Selden2020-ni**; **Shott2010-fn**;  
212 **Smith2015-qk**; **Thulman2012-fo**).

213 The project was initially designed to compare the EFA results with a semilandmark analysis using the full  
214 outline of the projectile point ~~by taking a sample of the outline and using the sampled coordinates as landmarks.~~  
215 [The projectile point outline consists of a series of coordinates describing the outline. Semilandmarks can](#)  
216 [be obtained by sampling the outlines to create an equal number of coordinates, which are then treated as](#)  
217 [semilandmarks.](#) Both approaches yielded similar results, but neither achieved satisfactory accuracy. The  
218 research design was then modified to include a more traditional landmark analysis to determine whether it

219 would improve upon the initial design.

220 There are some disadvantages to using landmarks, which is why the EFA/semilandmark approach was ini-  
221 tially favored. The principal disadvantages to using landmarks are reproducibility and accuracy. Landmarks  
222 are more subjective in many ways than the semilandmarks or EFA (**Shott2010-fn**). The analyst must decide  
223 how many points to place, what topological points should be used as landmarks, and how many landmarks  
224 should be used. The placement of landmarks can vary between analysts and can be affected by the instru-  
225 ments or software used to collect or create the landmarks. Another major concern is the loss of detail from  
226 not considering the entire outline. Serrated [projectile](#) points and points with more than one notch (this occurs  
227 more often than one might expect in Southwestern [projectile](#) points) are difficult to capture without including  
228 ~~a lot of many~~ landmarks, which are only applicable in a minority of situations. Secondary to these points, but  
229 still a concern, is that placing landmarks can be a more time-consuming process, as it is not as [easily](#) subject  
230 to automation as semilandmarks or EFA (**Palaniswamy2010-sl**).

231 Despite the disadvantages, landmarks are widely used for good reasons. I see two main advantages to  
232 landmark analysis in the context of projectile point analysis. The first is that the analyst can use their prior  
233 experience to determine what topological locations on the projectile point are most useful for discriminating  
234 between types. Decades of research on projectile points has refined many typologies into useful tools, despite  
235 their limitations. This knowledge can be applied to choosing appropriate landmarks. The second advantage is  
236 that outline analysis requires complete [projectile](#) points, whereas landmark analysis can use damaged points.  
237 If chunks of the projectile are missing then the outline is not usable. Possibly, the missing portion could be  
238 estimated and filled in, but that process is more error prone than estimating missing landmarks. Landmarks  
239 can be placed on reconstructed projectile point illustrations or missing landmarks can be estimated mathe-  
240 matically (**Gunz2009-yb**). Most projectile points suffer from some type of damage and some of the projectile  
241 points I classified as “complete” suffer from minor damage to the tip of the point or elsewhere. The use of  
242 damaged points can greatly increase the available sample size for studies, which is often a major limitation in  
243 projectile point studies.

244 Landmark configurations can vary significantly, depending on what the analysis is designed to measure and  
245 on the point type. Most [projectile point landmark analyses incorporate both landmarks and semilandmarks.](#)  
246 [The difference being that landmarks are placed on homologous points \(notches, corners, etc.\) and semilandmarks](#)  
247 [are placed equidistantly along a curve or line.](#) Most of the area of a projectile point is usually in the blade—the  
248 portion above the notches. The base of the point, the portion below the notches, is also the hafting element.  
249 For projectile point typologies, the base of the point usually contains the most important elements for deter-  
250 mining the type—notching style and basal shape being the two major elements. Thus, if most of the landmarks  
251 [or semilandmarks](#) are on the blade margins then the base of the point is not getting as much coverage. It is  
252 more than just tradition that the base gets the most attention. Hafting a point is an important technological  
253 choice, more so than how long the point is. Furthermore, projectile points can be resharpened. Resharpening  
254 the blade margins can modify the shape of the blade and change its appearance. While it is possible to modify  
255 the base of the point and even convert a side-notched point into a corner-notched point and vice-versa, it is un-  
256 likely that this happened regularly with the small arrowpoints used in this study (**Loendorf2019-df**). [Because it](#)  
257 [is necessary to incorporate different landmarking procedures for each projectile point shape, I separated the](#)  
258 [projectile points into three classes: side-notched, corner-notched, and triangular. For this study, I combined](#)  
259 [stemmed points into the corner-notched category.](#)

260 Because of the vagaries of placing landmarks, I used two configurations in this study. In the first configura-  
261 tion, the full outline was used. ~~Only~~ [In the second configuration, I used](#) what I term ~~the corner~~ the “corner”  
262 of the projectile point ~~was used in the second configuration. Figure 8.~~ [Figure 7](#) shows the first landmark con-  
263 figurations. [For simplicity, I will refer to both landmarks and semilandmarks in the following discussion as](#)  
264 [landmarks.](#) The landmark configuration was designed to place fewer landmarks along the blade margins and  
265 more landmarks along the notches and the base. Separate curves were placed between the tip of the point  
266 and the notches and the notches and the base (or the tip and the base for triangular [projectile](#) points), and

**Table 2.** Linear Discriminant Analysis Results for Projectile Point Shapes

<u>Shape</u>	<u>EFA</u>	<u>semiLdk</u>	<u>Mean</u>
<u>Corner-notched</u>	<u>0.88</u>	<u>0.91</u>	<u>0.90</u>
<u>Side-notched</u>	<u>0.65</u>	<u>0.74</u>	<u>0.70</u>
<u>Triangular</u>	<u>0.76</u>	<u>0.59</u>	<u>0.68</u>
<u>Mean</u>	<u>0.76</u>	<u>0.75</u>	<u>0.76</u>

landmarks were placed at equidistant locations along the curves. The second configuration is much sparser (figure 98). The landmarks were placed only on the right side of the ~~point-this-was-arbitrarily-chosen~~point. For the side-notched and corner-notched projectile points, landmarking started from the top of the notch, moved to the middle of the notch and then the bottom portion of the notch. For corner-notched points, this last landmark marked the right corner of the point, but for side-notched points an additional landmark was ~~placed~~needed to mark the base of the point. The final landmark was placed at the center of the basal margin. Triangular points differed by placing the first landmark in the center of the blade margin. The first approach contains between 30 and 42 landmarks that cover the entire point outline, whereas the second ranges from 3 to 5 landmarks that cover only a portion of the projectile point. These extremes were chosen to provide significant contrast between approaches.

## Results

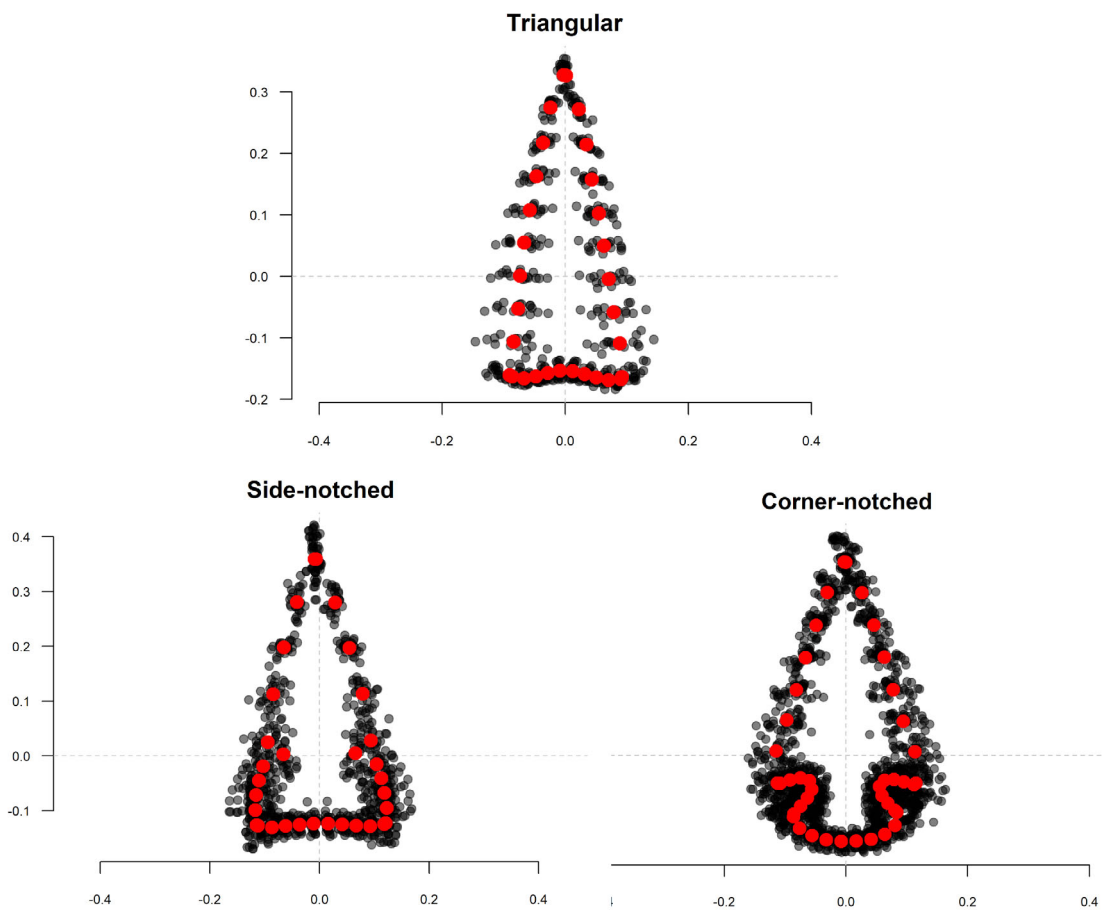
### Justice Projectile Points

The first step in the analysis was to determine how well projectile points typed by Justice could be correctly assigned using GM methods. Linear discriminant analysis (LDA) was used to type the projectile points using the GM results. A general target of 0.85 was arbitrarily chosen as a minimum target for acceptable results—meaning that 85% of the projectile points were classified correctly. As mentioned previously, Justice placed each projectile point type into a cluster. Presumably, projectile point types in the same cluster should be more closely related than they are to projectile point types in other clusters. This gives another level of comparison that was used in addition to the types.

The original intent was to compare EFA versus semilandmarks placed at equidistant locations around the outline. However, these results were unsatisfactory, and a more traditional landmark analysis was also completed. Because the number and placement of landmarks has a significant impact on the outcome of the study, two different landmark configurations were used. Tables 2, 3, and 4 show the LDA results by type, cluster, and by shape—the column and row means are also included. These tables will be referred to in the sections that follow.

Part of the reason the results were unsatisfactory for the EFA and semilandmarks was that the LDA analysis had trouble discriminating between notched and unnotched projectile points and between side-notched and corner-notched points. These are some of the most basic distinctions that are made when analyzing projectile points. While it would be convenient if the analysis did not require an additional step, it is not difficult to separate the projectile points into these basic shapes prior to the GM analysis. ~~I separated the Justice points into three classes: Table 2 shows the accuracy of EFA and semilandmark analysis for identifying each type of shape. The landmark analysis was not compared as each shape used a different landmark configuration. The primary challenge was identifying~~ side-notched ~~, corner-notched, and triangular. For this study, I combined stemmed points into the corner-notched category. from triangular points.~~

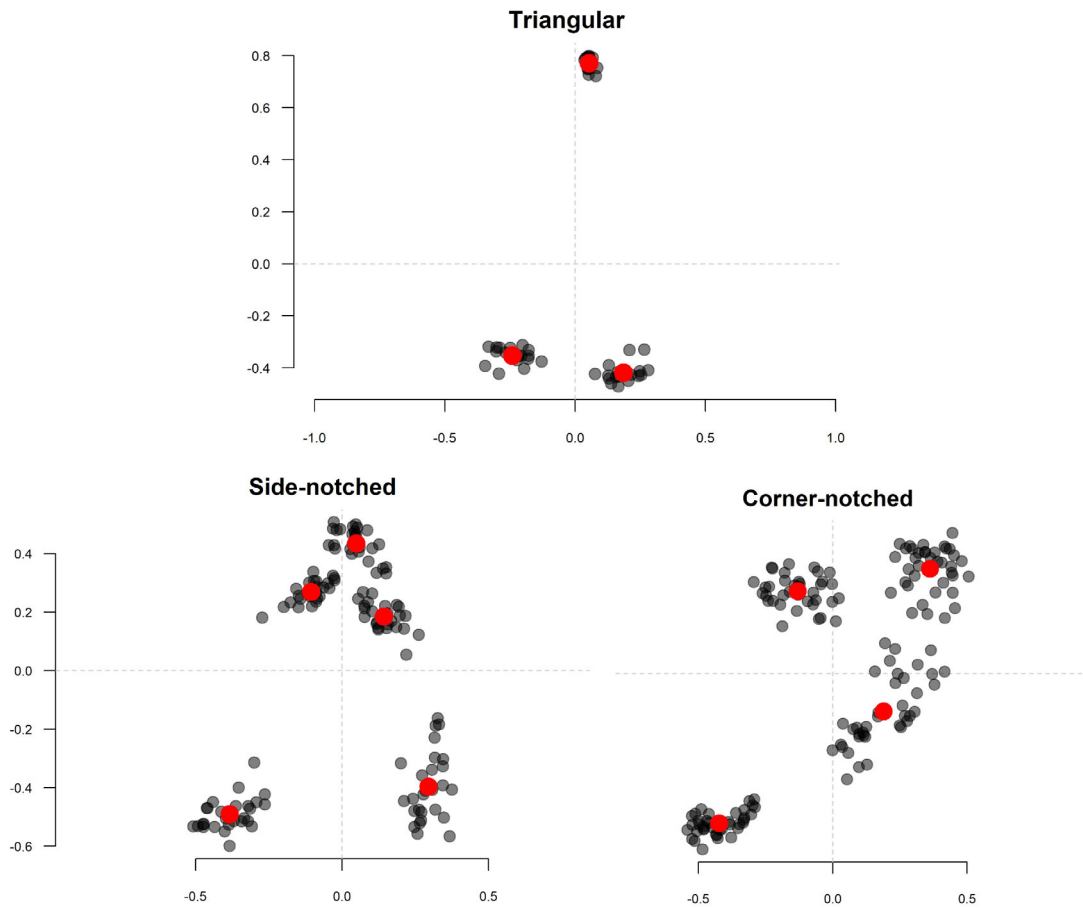
~~Tables 2, 3, and 4 show the LDA results by type, cluster, and by shape. These tables will be referred to in the sections that follow.~~



**Figure 7.** Comparison of the full outline landmarks for corner-notched, side-notched and triangular shaped [projectile](#) points from Justice's (2002) projectile point illustrations. Red dots indicate the mean location for each of the landmarks.

**Table 3.** Linear Discriminant Analysis Results for Projectile Point Types

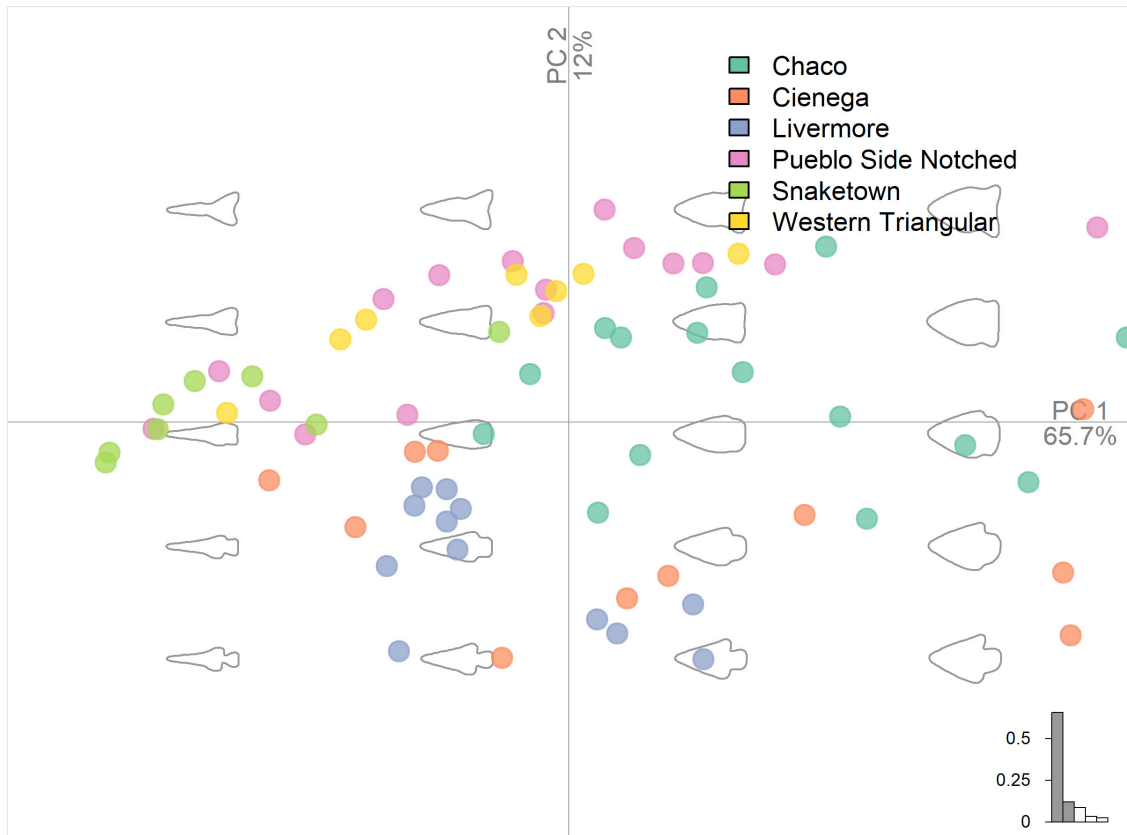
Type	EFA	semiLdk	Ldk	Ldk-corner	Mean
Chaco Corner Notched	0.56	0.89	0.85	0.85	0.79
Cottonwood Triangular	0.38	0.38	1.00	0.75	0.63
Guadalupe	0.75	0.83	0.93	0.93	0.86
Pueblo Alto Side Notched	0.86	0.71	1.00	1.00	0.89
Pueblo Side Notched Concave Base	0.44	0.78	0.73	0.73	0.67
Pueblo Side Notched Straight Base	0.57	0.57	0.43	0.71	0.57
Snaketown Triangular Concave Base	0.78	0.89	0.78	0.89	0.84
Tularosa Corner Notched	0.77	0.69	0.60	0.80	0.72
Mean	0.64	0.72	0.79	0.83	0.74



**Figure 8.** Comparison of the projectile point corner's landmarks for corner-notched, side-notched and triangular shaped points from Justice's (2002) projectile point illustrations. Red dots indicate the mean location for each of the landmarks.

**Table 4.** Linear Discriminant Analysis Results for Projectile Point Clusters

Cluster	EFA	semiLdk	Ldk	Ldk-corner	Mean
Chaco	0.81	0.88	0.92	0.92	0.88
Cienega	0.69	0.69	0.60	0.80	0.70
Livermore	0.83	0.75	0.93	0.93	0.86
Pueblo Side Notched	0.75	0.56	0.89	1.00	0.80
Snaketown	0.78	0.89	0.78	0.89	0.84
Western Triangular	0.38	0.50	1.00	0.75	0.66
Mean	0.71	0.71	0.85	0.88	0.79



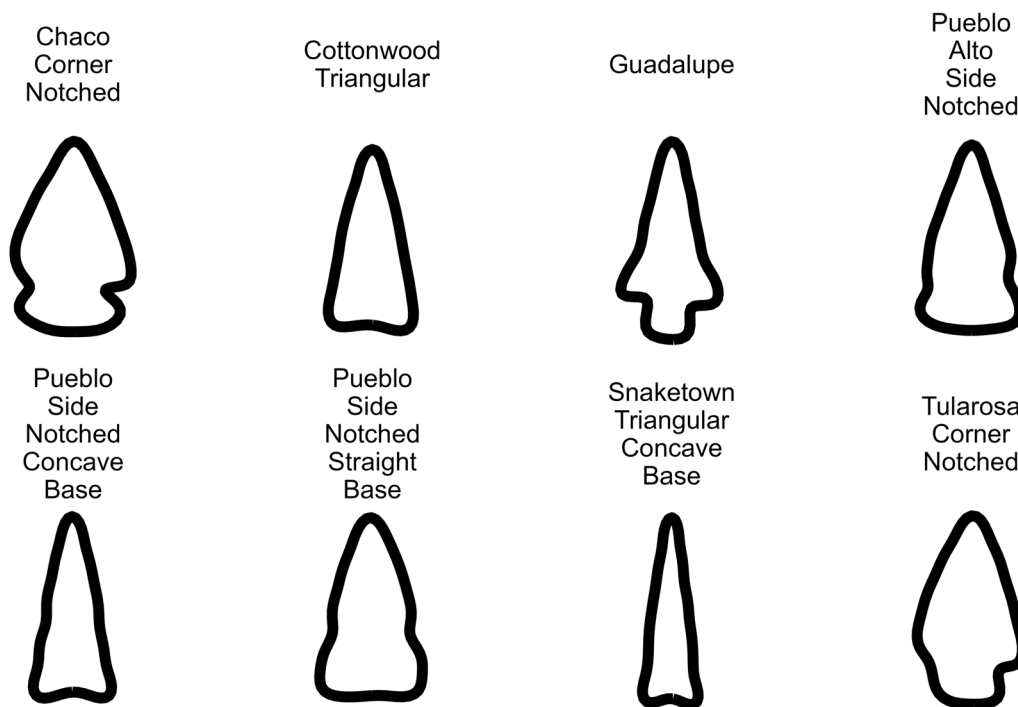
**Figure 9.** Principal components plot showing projectile points from Justice (2002) and the morphospace based on an elliptical Fourier analysis. The [projectile](#) points are labeled by the cluster assigned by Justice.

303 **Elliptical Fourier Analysis**

304 [Linear Discriminant Analysis Results for Projectile Point Shapes Shape EFA semiLdk MeanCorner-notched-0.88](#)  
 305 [0.91-0.90Side-notched-0.65-0.74-0.70Triangular-0.76-0.59-0.68Mean-0.76-0.75-0.76](#)

306 The EFA analysis was conducted using the Momocs packages ([Bonhomme2014-gt](#)) in R ([R\\_Core\\_Team2022-wb](#)).  
 307 The first step was to calculate the number of harmonics to use. In this case 12 harmonics described 99% of the  
 308 variation in the projectile point outlines. Next, the EFA function was used on each projectile point, and then  
 309 a PCA was used to reduce the dimensionality [-Figure 10 of the data](#). [Figure 9](#) shows the results of the PCA  
 310 analysis. A useful feature of PCA plots using these data is that the morphospace can be plotted with the PCA  
 311 results. The morphospace shows how the shapes vary along each axis of the PCA. In this case, PC1 (the first  
 312 principal component) varies between short, wide [projectile](#) points and long, narrow points. Some of shapes  
 313 on the top and bottom left have inverted into impossible shapes, but note that no [projectile](#) points fall into  
 314 these areas. PC2 varies primarily from stemmed points to side-notched [projectile](#) points.

315 The first objective for the analysis of the Justice [projectile](#) points is to determine how well the different  
 316 point types can be discriminated. Meaning, how well can GM methods classify these [projectile](#) points into  
 317 their original categories. The LDA results were far from the target goal of 0.85 for most projectile point types,  
 318 and only one type (Pueblo Alto Side [NotchedNotched-0.86](#)) met the target (see EFA results in [Table 23](#)). The  
 319 [LDA](#) results were better when the projectile point types were grouped into clusters, as shown in [Table 34](#);  
 320 however, none of the clusters met the target of 0.85. Even more disconcerting were the results shown in  
 321 [Table 42](#), as only the corner-notched [projectile](#) points were discriminated with an accuracy greater than the  
 322 target.



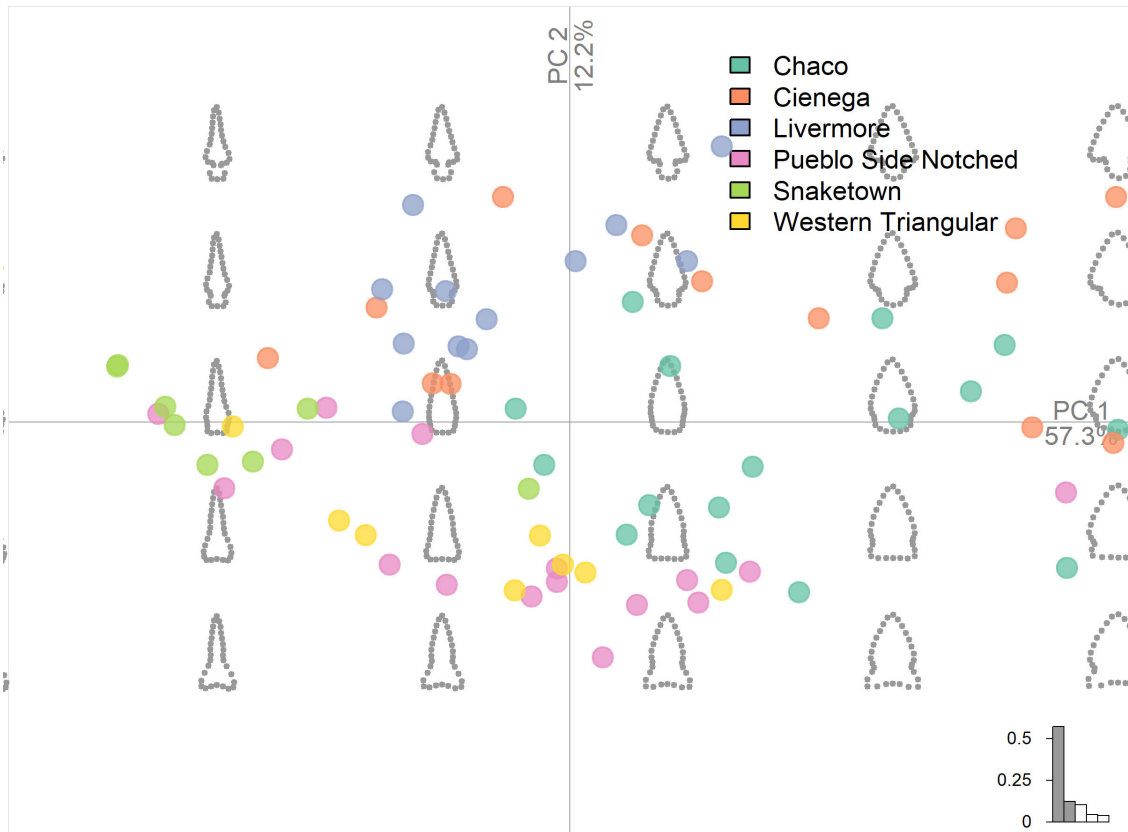
**Figure 10.** Mean shapes by projectile point type using elliptical Fourier analysis.

323 The differences in classification accuracy between corner-notched, side-notched, and triangular [projectile](#)  
 324 points can perhaps best be explained by examining the mean shapes of each point. [Figure 11](#), [as generated](#)  
 325 [through EFA](#). [Figure 10](#) shows the mean shapes for the selected projectile point types. These are the mathe-  
 326 matically average shapes when all of the projectile points in the type are combined. This has a tendency to  
 327 average out the notches for the side-notched points, as the placement of these notches vary in height. Pueblo  
 328 Alto Side Notched points appear to be an exception to the side-notched problem, as they have the highest  
 329 classification accuracy. Corner notched and stemmed points must, by definition, always have their notches or  
 330 stems in the same location, even though the shape of the notches and stems still varies. This explains why it is  
 331 easier to discriminate them from other point types. As for the side-notched and triangular points, sometimes  
 332 the notches are subtle and the notches are only a small part of the whole form, which is clearly not a strong  
 333 enough element to separate triangular and side-notched points consistently.

### 334 Semilandmarks

335 Analyzing projectile points using semilandmarks is comparable to the EFA analysis. The major analytical choice  
 336 is how many landmarks to use. Each projectile point is represented by a varied number of coordinates that  
 337 represent its outline. The number of points must be standardized so that each projectile point has an identical  
 338 number of coordinates, which is done using the Momocs package. The choice of how many points to use does  
 339 affect the GM analysis. To solve this problem, I tried different numbers of semilandmarks varying from 10 to  
 340 100. More than 100 points appears to no longer have a substantial effect on the results. Each projectile  
 341 point was sampled multiple times and then all of the points were classified using LDA according to the same  
 342 procedures used for the EFA analysis. The results of this test ranged from 65% accuracy (10 points) to 73%





**Figure 11.** Principal Components Plots showing projectile points from Justice (2002) and the morphospace based on a semilandmark generalized procrustes alignment. The [projectile](#) points are labeled by the cluster assigned by Justice.

343 accuracy (30 points—the number used for the final analysis) with [somewhat higher numbers of points](#) [points](#)  
 344 [higher than 30](#) consistently measuring in at 64% accuracy.

345 The [morphospace](#) (see figure 12) based on the PCA analysis has [PCA components](#) had similar dimensions  
 346 of variation as the EFA analysis (see figure 11). The major improvement in accuracy (the ['semiLdk'](#)  
 347 column of Tables 2-4) is perhaps due to the better alignment generated by the GPA procedure, but that is only  
 348 speculation.

349 Regardless of the reason, a jump from 64% classification to 72% [between the EFA and semilandmark](#)  
 350 [analysis](#) is a substantial improvement. It does not reach the target of greater than 85% classification accu-  
 351 racy, but it is a step in the right direction. The mean shapes are nearly identical to the EFA analysis, which  
 352 indicates that this method suffers from the same problems with side-notched and triangular [projectile](#)  
 353 points, but it does a somewhat better job differentiating corner-notched and side-notched points. Curiously, it does  
 354 a worse job differentiating triangular points. [It seems the](#) [The](#) side-notches are [still a problem](#) [the likely culprit](#).

### 355 Landmarks

356 Neither the semilandmarks nor EFA adequately distinguished between point types or between point shapes.  
 357 Identifying notches was particularly troublesome. A solution to this problem was to use landmarks and ex-  
 358 plicitly identify the notches or lack of notches. No comparison was made between projectile point shapes  
 359 (i.e., [triangular versus side-notched](#)) using landmark analysis, as initial experiments determined that it was  
 360 best to use different landmark procedures for the different shapes. Perhaps machine learning may solve

361 this problem ([Castillo\\_Flores2019-cs](#); [MacLeod2018-aj](#); [Nash2016-mc](#)). Triangular [projectile](#) points require  
362 a different approach than side-notched points, and even side-notched and corner-notched/stemmed points  
363 require different procedures.

364 The LDA results for the first landmark configuration (Ldk in Tables [2-43-4](#)) are much better than EFA and  
365 better than the semilandmark analysis, but still not as accurate as desired. The biggest underperformer by far  
366 was Pueblo Side Notched Straight Base at 0.43. All of the previous analyses struggled to capture basal shape  
367 distinctions, but this analysis struggled more so, [as the base is a critical component of this type](#). What is par-  
368 ticularly notable is that Cottonwood Triangular [projectile](#) points were classified perfectly whereas they were  
369 previously the worst performing type [in the EFA and semilandmark analyses](#). As Table [3-4](#) shows, the clus-  
370 ter assignments performed well. If ~~the problems sorting Cienega from Snaketown can be sorted out~~[Cienega](#)  
371 [points were not so problematic](#), then the results would be excellent.

372 The final analysis used the second landmark configuration—the projectile point corners. These results  
373 proved superior to the first landmark configuration and are almost 20% [points](#)-higher in accuracy than the EFA  
374 results on average. The lowest type for accuracy was again Pueblo Side Notched Straight Base, but it improved  
375 from the first landmark configuration to 0.71 from 0.43. The accuracy results were more consistent and accu-  
376 rate. With some [enhancement](#)[additional experimentation on landmark placement](#), this configuration could  
377 likely achieve better results [and meet the targeted 0.85 accuracy](#).

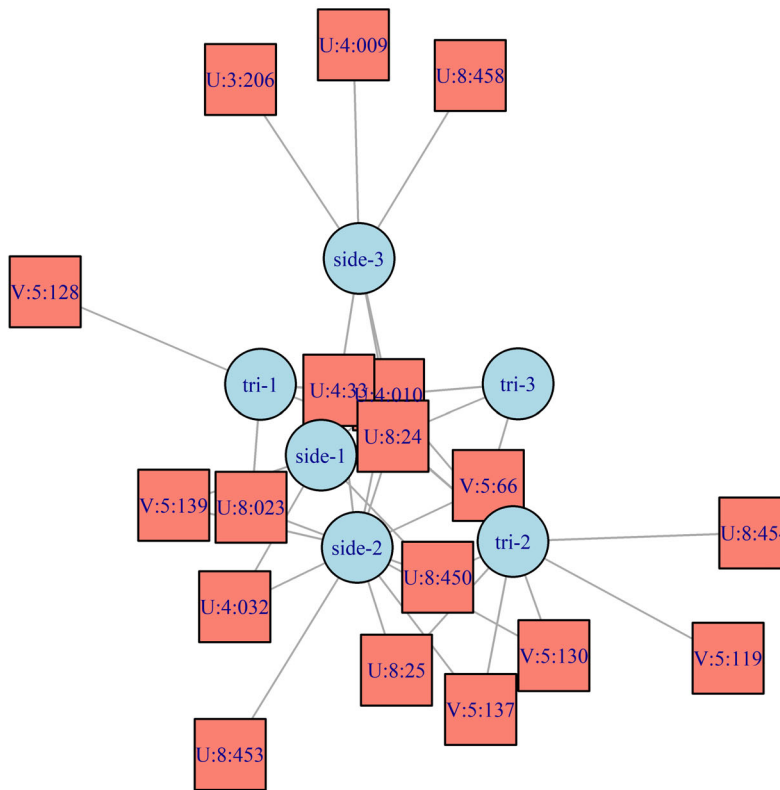
378 Not only did landmark analysis provide superior accuracy, but it will also make it easier to use larger sample  
379 sizes. Presumably, notching style is an important attribute that should be captured in the analysis. If EFA or  
380 the semilandmark analysis as conducted here fails to sufficiently emphasize the notches, then these methods  
381 are insufficient for [my purposes](#)[classifying projectile points](#). While landmark analysis is more time-consuming,  
382 the use of the second configuration does reduce the burden of landmarking.

## 383 **Tonto Basin Projectile Points**

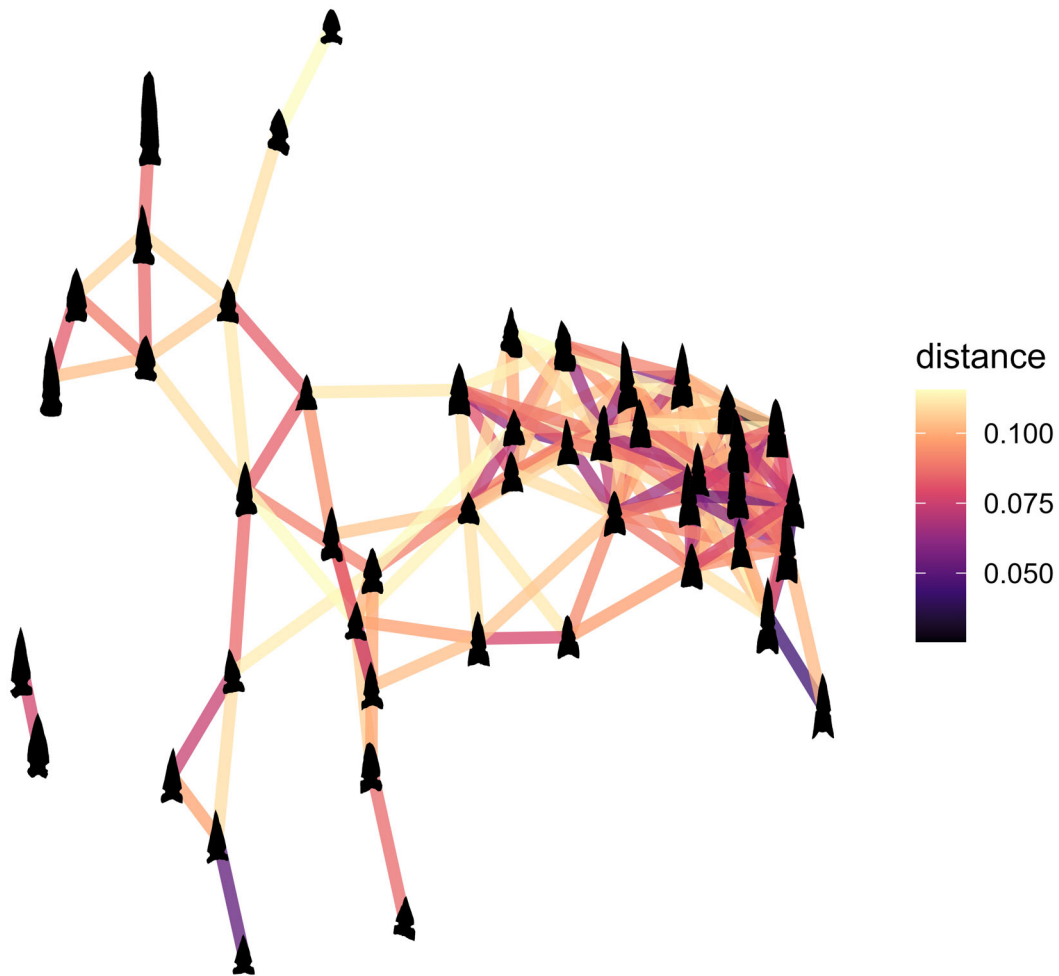
384 The initial intent was to classify the Tonto Basin [projectile](#) points using the analysis of the Justice [projectile](#)  
385 points; however, the limited sample size limits the validity of the exercise. The analysis was not futile though, as  
386 the second landmark configuration using the corners of the projectile points proved the most effective. I there-  
387 fore used the same landmark configuration to analyze the [projectile](#) points from Tonto Basin. The results of  
388 the GPA and PCA analysis were used in a hierarchical cluster analysis using Ward's method ([Murtagh2014-mb](#)).  
389 Figure [13-12](#) is a network graph showing the results. [This graph shows the assigned projectile point types from](#)  
390 [the cluster analysis with connections from every Tonto Basin site where that projectile point was found to the](#)  
391 [assigned type](#). Several sites in Tonto Basin only had one or two types of projectile points (low sample sizes  
392 were again problematic), but some of the larger, well-excavated sites shared all or most of the projectile point  
393 types. It is beyond the purpose of this study to explore the patterns in this data, but the methods clearly  
394 provide useful data for exploratory analysis.

395 The final question I wished to address in this study was whether it was necessary to use a typology in a GM  
396 analysis of projectile points. There are many ways to answer this question, but in short, the answer is no. That  
397 does not mean typologies are not useful, but they can mask important variation. The following is one way to  
398 approach analyzing projectile points without using a typology.

399 Because the results of the GM analyses can be projected into multidimensional space, the distance between  
400 these values is meaningful and can be directly compared. One way to flatten multidimensional space [into two](#)  
401 [dimensions](#) is to calculate the Euclidean distance between each point and display the results as a network  
402 graph, as in figures [14 and 15](#). [13 and 14](#). This way each point can be compared directly without grouping the  
403 [projectile](#) points into types. The results are messier [than neatly fitting each point into a single type](#), but subtle  
404 variation in morphology is easier to visualize this way. The [closer the results should only be interpreted as a](#)  
405 [visual aid. The closer the projectile](#) points are to each other, the more similar they are in shape, keeping in  
406 mind that only the corners of the projectile points (from the notches down for the side-notched points) were  
407 used in this analysis. Many of the points clustered closely together, indicating a common shape across the



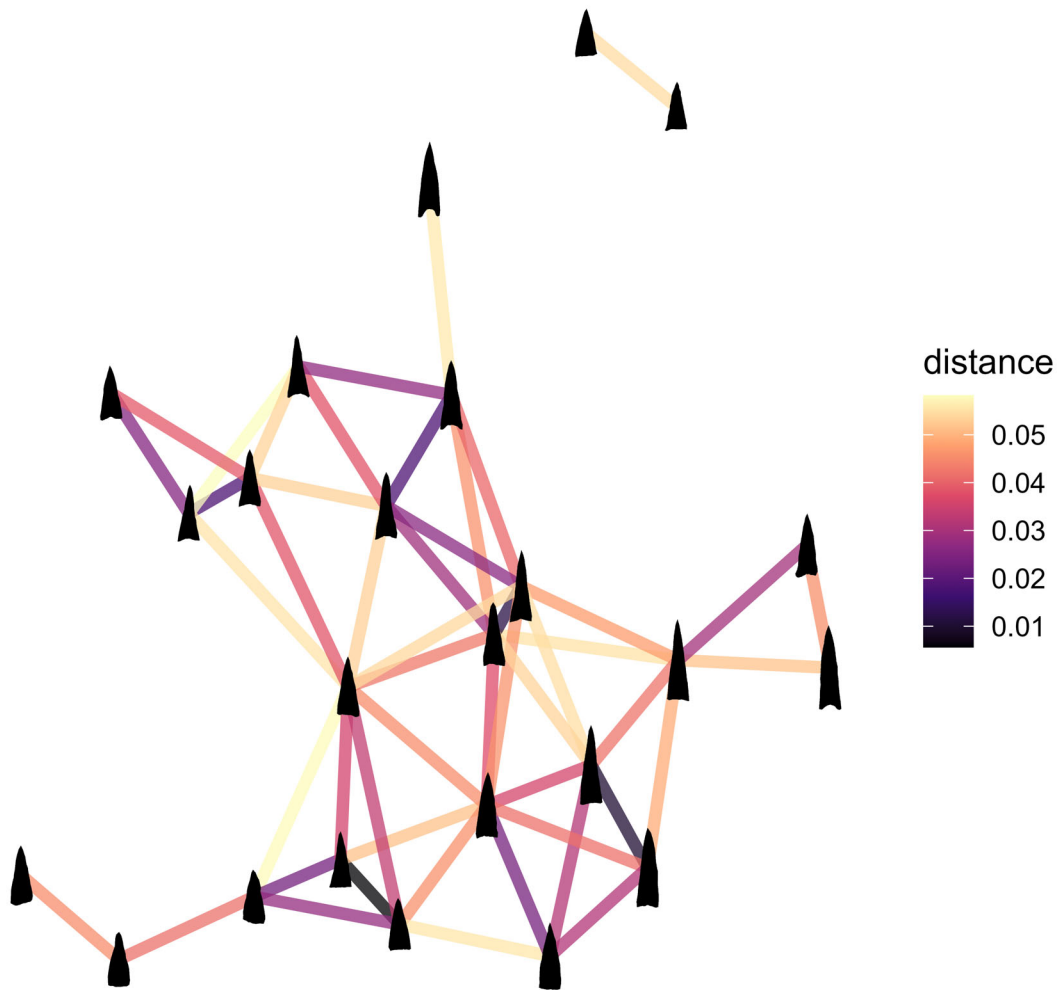
**Figure 12.** Bipartite network graph displaying assigned projectile point clusters for side-notched and triangular projectile points in Tonto Basin and Tonto Basin sites. The circles represent cluster designations by shape (e.g., side = side-notched and tri = triangular). The squares represent sites. The links between squares and circles show which point clusters are found at which sites.



**Figure 13.** Network graph displaying side-notched [projectile](#) points from Tonto Basin as nodes with ties showing the morphometric distance ([euclidean distance in Principal Component space](#)) between [projectile](#) points. Darker colors represent stronger ties. Note that only the strongest 10% of ties are shown.

408 sites. The side-notched points have a particularly large cluster of typical Hohokam side-notched points. Yet  
 409 there are also a large number of [projectile](#) points that do not closely match the [other](#) points, which indicates  
 410 that there is also a lot of variation. This variation may represent idiosyncracies, exchange, migration, novice  
 411 knappers, or a different intention for the point. Regardless of the purpose, the GM analysis better captures the  
 412 “otherness” of a [projectile](#) point than classifying a point as other/unknown or worse, forcing it into a category  
 413 it does not belong in.

414 The clustering analysis provided several different projectile point types and provided an overview of what  
 415 sites shared similar projectile points. This type of analysis can be combined with architectural or other data  
 416 to look for correlations or patterns that provide insights into the behavior of the people who made and used  
 417 these [pointsprojectiles](#). Yet the analyst must take care to ensure the clustering groups are appropriately sized  
 418 and the results make sense. One way to view the data more closely is to look at a distance network graph to  
 419 view the variation in morphometric shape. This way typological distinctions will not mask the variation.



**Figure 14.** Network graph displaying triangular [projectile](#) points from Tonto Basin as nodes with ties showing the morphometric distance ([euclidean distance in Principal Component space](#)) between [projectile](#) points. Darker colors represent stronger ties. Note that only the strongest 10% of ties are shown.

## 420 Conclusion

421 [The purpose of this paper was to evaluate geometric morphometric methods for analyzing projectile points in](#)  
422 [the Southwest U.S.](#) These analyses provided significant variation in their results, yet they also demonstrated  
423 positive results. While EFA underperformed all of the other analyses, a different dataset may favor this analysis.  
424 Some of the types performed better for EFA than other types, which suggests that EFA may be the optimal  
425 choice for some datasets. [Indeed a recent case study found that EFA performed comparable to or better](#)  
426 [than landmark analysis in several case studies \(Matzig2021-id\)](#). A clear result from this exploratory analysis is  
427 that GM analysis is not a one-size-fits-all approach. [Better results were obtained from a full outline approach](#)  
428 [using semilandmarks, which raises interesting theoretical questions I am unable to address here but would](#)  
429 [be worthwhile to pursue further. More traditional landmark approaches performed better, likely because](#)  
430 [the outline approaches failed to identify side-notches consistently.](#) In this case, [a landmark/semilandmark](#)  
431 [method](#) using the corner of the projectile point—from the base to the middle of the basal margin or from the  
432 middle of the blade if the point is triangular—proved to be the most useful method. The main advantage of this  
433 method was that it provided the most accurate reproduction of Justice’s original classification of the projectile  
434 points. Another advantage is that broken points are easier to use with this method. If one half of the point  
435 is missing, either the top or the lateral margin, it does not affect the analysis. This increases the number of  
436 points available for analysis tremendously compared to only using whole points. The final advantage, though  
437 minor, is that the landmarking analysis is simple and only requires three to five landmarks.

438 I mentioned ~~in the introduction~~ how difficult it is to conduct regional analyses with projectile points in the  
439 U.S. Southwest. The main difficulty is harmonizing existing typologies and then fitting new projectile points  
440 into this typology. While the sample size available for this study was too small to attempt classifying the  
441 Tonto Basin points according to Justice’s typology, it would be possible to do so with these methods [given](#)  
442 [enough data](#). However, this ~~chapter paper~~ also demonstrated that it is possible to type [projectile](#) points us-  
443 ing common clustering methods which may better capture the variation in projectile point morphology than  
444 previously used types. Furthermore, it is possible to analyze projectile points without resorting to types. The  
445 distances between projectile point morphologies can be computed and compared directly. These distances  
446 could even be aggregated and summarized regionally. The main challenge for the regional analysis is obtain-  
447 ing the projectile point outlines or landmarks. Once these are obtained, thousands of points can be analyzed  
448 and assigned to clusters relatively quickly.

449 Compared to a traditional analysis of linear metrics and weights, a GM analysis can capture much more in-  
450 formation and provide more informative ways to analyze and visualize the data. The visualization capabilities  
451 of GM is one of its greatest strengths, as it allows the analyst to see the data they are working with, visually  
452 validate their results, and share their findings in visually compelling ways. Additionally, this analysis is more  
453 reproducible and adaptable than traditional lithic analyses. While the analyst still has a lot of control over a  
454 GM analysis, the results should be less biased than analyses based on visual type comparisons.

## 455 Acknowledgements

456 Special thanks to Zac Selden, Michael Shott, Bernard Means, and Loren Davis for teaching a geometric mor-  
457 phometric workshop that was my first major introduction to these methods. ~~Josh Watts, Matt Joshua Watts,~~  
458 [Matthew](#) Peeples, Melissa Powell, ~~Chris Christopher~~ Caseldine, and several volunteers ~~who~~ assisted with this  
459 research.

## 460 Data, scripts, and supplementary information availability

461 All relevant data and scripts are available at the following [DOI 10.17605/OSF.IO/ZGE9Q](https://doi.org/10.17605/OSF.IO/ZGE9Q) and on GitHub at  
462 [github.com/bischrob/TontoBasinPoints](https://github.com/bischrob/TontoBasinPoints). The [R](#) code used in the analysis is included in the manuscript.Rmd

463 file used to create this manuscript, although some lines have been commented out to improve efficiency.

## 464 **Conflicts of interest disclosure**

465 The author declares that they comply with the PCI rule of having no financial conflicts of interest in relation to  
466 the content of the article.